Chapter 12
Surface Area and Volume

Section 4
Volume of Prisms and Cylinders

The **volume of a solid** is the number of cubic units contained in its interior. Volume is measured in cubic units, such as cubic meters (m^3).

VOLUME POSTULATES

POSTULATE 27 ***Volume of a Cube***

The volume of a cube is the cube of the length of its side, or $V = s^3$.

POSTULATE 28 ***Volume Congruence Postulate***

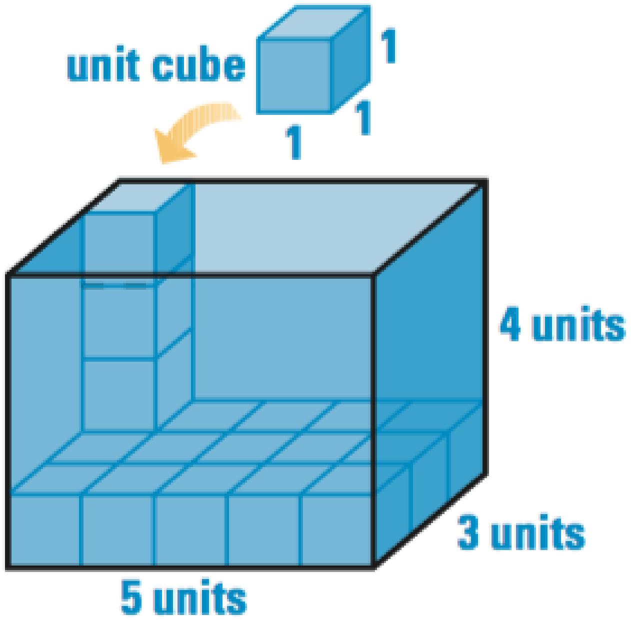
If two polyhedra are congruent, then they have the same volume.

POSTULATE 29 ***Volume Addition Postulate***

The volume of a solid is the sum of the volumes of all its nonoverlapping parts.

Example 1: Finding the Volume of a Rectangular Prism

The box shown is 5 units long, 3 units wide, and 4 units high. How many unit cubes will fit in the box? What is the volume of the box?



15
15
15
15

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60 units^3

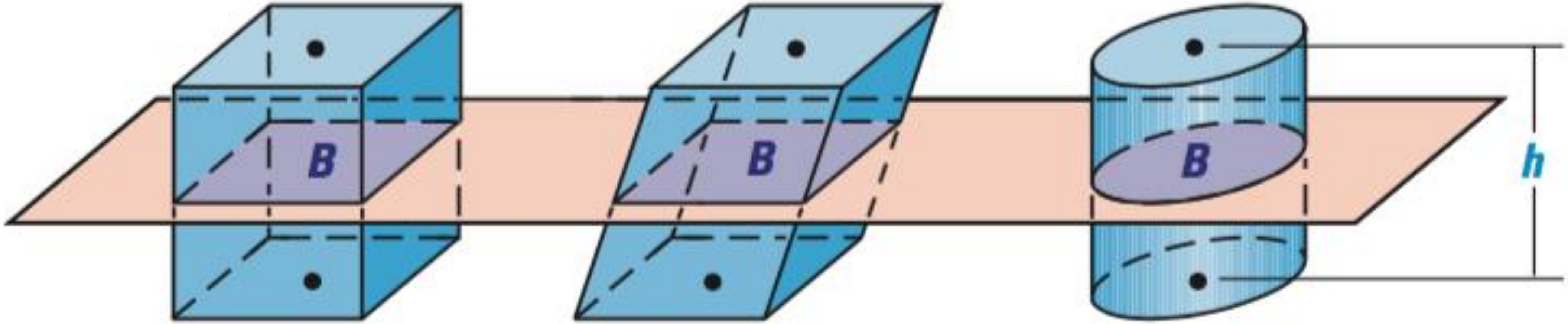
GOAL 2: Finding Volume of Prisms and Cylinders

THEOREM

THEOREM 12.6 ***Cavalieri's Principle***

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

Theorem 12.6 is named after mathematician Bonaventura Cavalieri (1598–1647). To see how it can be applied, consider the solids below. All three have cross sections with equal areas, B, and all three have equal heights, h. By Cavalieri's Principle, it follows that each solid has the same volume.



Rect: $l \times w \times h$

Tri.: $(\frac{1}{2} b h) h$ $\left(\frac{1}{2} \times l \times w \times h\right)$

height of $\triangle$

height of prism

VOLUME THEOREMS

THEOREM 12.7 ***Volume of a Prism***

The volume V of a prism is $V = Bh$, where B is the area of a base and h is the height.

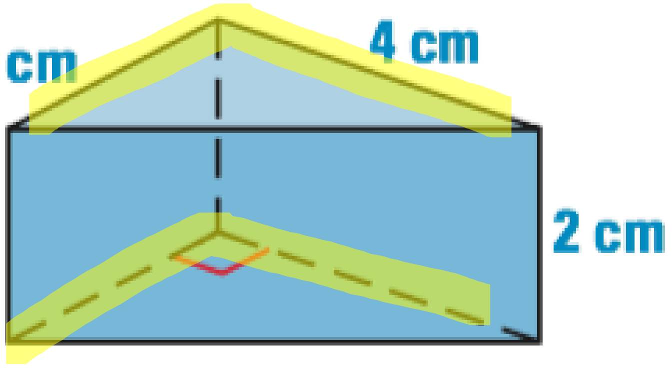
THEOREM 12.8 ***Volume of a Cylinder***

The volume V of a cylinder is $V = Bh = \pi r^2 h$, where B is the area of a base, h is the height, and r is the radius of a base.

Example 2: Finding Volumes

Find the volume of the right prism and the right cylinder.

a.

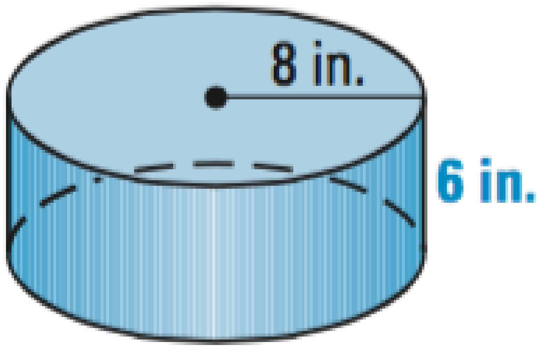


$(\frac{1}{2}bh)h$

$(\frac{1}{2} \times 3 \times 4)2$

12 cm^3

b.



$\pi r^2 h$

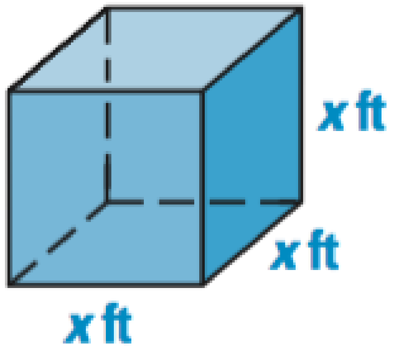
$(3.14)(8^2)(6)$

1205.76 in^3

Example 3: Using Volumes

Use the measurements given to solve for x.

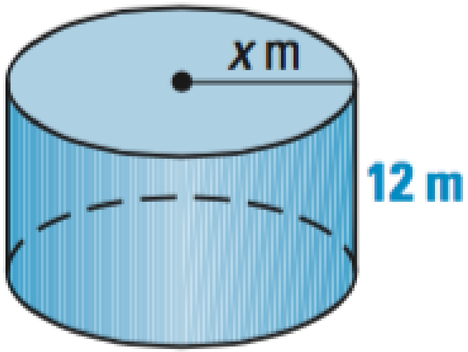
a. Cube, $V = 100 \text{ ft}^3$



$\cancel{\sqrt[3]{x^{3}}} = \sqrt[3]{100}$ $\left(100^{1/3}\right)$

$x = 4.64 \text{ ft}$

b. Right cylinder, $V = 4561 \text{ m}^3$



$4561 = 3.14(x^2)(12)$

$\frac{4561}{37.68} = \frac{\cancel{37.68}x^2}{\cancel{37.68}}$

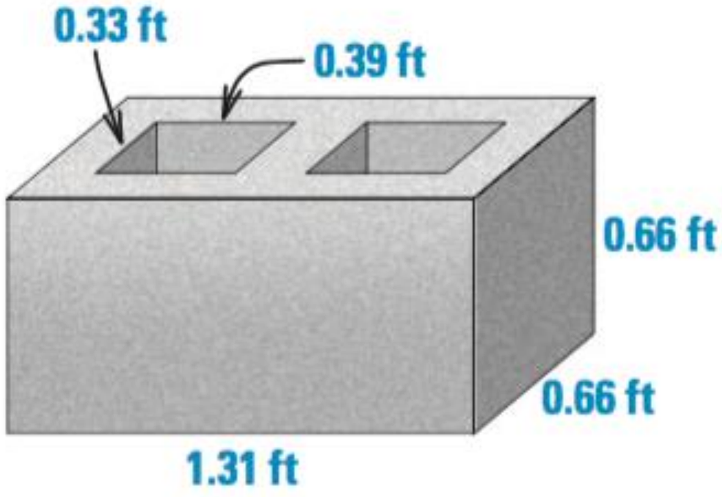
$\sqrt{121.05} = \sqrt{x^2}$

$11 \text{ m} = x$

Example 4: Using Volumes in Real Life

CONSTRUCTION Concrete weighs 145 pounds per cubic foot. To find the weight of the concrete block shown, you need to find its volume. The area of the base can be found as follows:

$$B = \boxed{\text{Area of large rectangle}} - 2 \cdot \boxed{\text{Area of small rectangle}}$$



Volume of large: 1.31 x .66 x .66 = .570636 ft^3

Volume of hole: .33 x .39 x .66 =.084942 x 2 = .169884 ft^3

Cinderblock: .570636 - .169884 = .400752 ft^3

→ Weight: .400752 x 145 = 58.1 pounds

EXIT SLIP